Induction versus Deduction in Science, Computing, Literature and Art

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Induction and deduction are important cognitive processes, which exist in all spheres of human culture. These processes are analyzed, formalized and utilized in logic and mathematics. However, to better understand their essence, we examine these processes using theoretical tools of computer science and describing their role in science, computing, literature and art.

As a cognitive mechanism, *deduction* is a type of logical inference of knowledge performed by application of specific deduction rules having, in general, the form

$$A \rightarrow B$$

or

$$A \vdash B$$

where A is called the *assumption* of the rule, B is called the *conclusion* of the rule, and both of them consist of a finite number of expressions, formulas or statements.

For instance, taking the expression "*X* is equal to *Y*" with variables *X* and *Y*, we can build the formal deduction rule

"*U* is equal to *V*"& "*V* is equal to *W*" ⊢ "*U* is equal to *W*"

It means

If *U* is equal to *V* and *V* is equal to *W*, then *U* is equal to *W*.

Recursive algorithms and the majority of logical systems formalize deduction, which became the basic inference tool in logic and mathematics.[2] In mathematics, the term *deduction* is often used as a synonym of the term *proof*.

At the same time, mathematical and scientific practice shows that deduction is used not so much for knowledge production as for knowledge justification because, as Aristotle observed, scientific discovery by deduction is impossible, except one knows the "first" primary premises, and it is necessary to obtain these premises by induction.[3]

As a cognitive mechanism, *induction* is a form of logical inference that allows inferring a general statement from a sufficient number of particular cases, which provide *evidence* for the general statement induced (the conclusion). However, if the evidence is not complete, the conclusion may be incorrect. For instance, Aristotle saw that all of the swans in the places he lived were white, so he induced that all swans are white in general. However, much later, Europeans came to Australia and discovered black swans. This shows that in contrast to deduction, induction does not always give correct results because it works with incomplete information, while the number of initial cases usually is not bounded and the researcher does not know for sure when to stop. However, the whole science is actually built on induction because scientific laws have to be in agreement with nature for natural sciences and with social systems for social sciences, while it is possible to make only a finite number of experiments.

Naturally, there is also inductive learning, which involves making uncertain inferences that go beyond direct experience and are based on intuition and insight.

However, this is only one kind of induction – *empirical induction*, correct application of which demands highly developed intuition and can be invalidated by some new observations or experiments. There is also *mathematical induction*.

Mathematicians, being largely dissatisfied by absence of absolute reliability in empirical induction used by physicists and other scientists, elaborated mathematical induction, which, in essence, reduces induction to deduction by the *axiom of induction*, which is very popular in mathematics and has several forms.

While mathematical induction eliminates necessity of intuition in making the conclusion, acceptance of the axiom of induction and its application still demand intuition.[4]

Interestingly, the progress in computer science and mathematics was achieved by going from the models of computation based on deduction, such as *Turing machines*, to the models of computation based on empirical (scientific) induction, such as *inductive Turing machines*.

Let us consider these models.

In his pioneering paper published in 1936, Turing clearly explains that his a-machine, later called Turing machine, mathematically models the work of a human computer. We can also add that it models the work of an accountant. In both cases, the goal of the process is computation of values of functions according to exact (mechanical) rules and stopping when the result is obtained.

> *"The idea behind digital computers may be explained by saying that these machines are intended to carry out any operations which could be done by a human computer. The human computer is supposed to be following fixed rules; he has no authority to deviate from them in any detail. We may suppose that these rules are supplied in a book, which is altered whenever he is put on to a new job. He has also an unlimited supply of paper on which he does his calculations. He may also do his multiplications and additions on a 'desk machine', but this is not important."*[5]

The work of a scientist or mathematician is essentially different because in essence, it is exploration, which can include calculation but cannot be reduced to it. This situation is reflected in the status of scientific theories and laws in general and the theories and laws of physics, in particular. For instance, Stephen Hawking writes:

> *"Any physical theory is always provisional, in the sense that it is only a hypothesis: you can never prove it. No matter how many times the results of experiments agree with some theory, you can never be sure that the next time the result will not contradict the theory."*[6]

That is why, in science, *e.g.,* in physics or biology, we can observe the following process of research.

First, scientist learn something about results of other scientists in their area.

Second (this step is sometimes skipped), scientists conduct some experiments and collect experimental data.

Third, scientists elaborate a hypothesis *L* often formulating it in mathematical terms.

Fourth, scientists conduct new experiments and check the hypothesis *L*.

Fifth, if a sufficient number of experiments support their hypothesis, scientists call *L* a law of nature.

Note that experiments are not only physical but also mental. Mental experiments are especially popular in mathematics.

As the time goes, the following situations are possible.

(a) Whenever all further experiments related to the law *L* support *L*, this law *L* is accepted forever as a law of nature.

(b) A new experiment contradicts *L*. In this case, either it is declared that *L* is not a law of nature or it is assumed that *L* is not valid in the initial domain. In both cases, *L* is rejected as a law of nature (either completely or for the initial domain) and scientists start searching for the new law, which more correctly than *L* describes the experimental data.

We can see that this process exactly reflects how an inductive Turing machine is functioning.[7] Note that if inductive Turing machines obtain their results, they do this in finite time, *i.e.*, making only a finite number of steps. They mathematically describe and formalize empirical induction, that is, inductive reasoning prevalent in science and mathematics. Note that induction used in science is not mathematical induction, which reduces induction to deduction.

Thus, we come to the following conclusion:

Turing machine formalizes the work of an accountant or a human computer.
Inductive Turing machine formalizes the work of a scientist and functioning of science.

Creative work of scientists, which includes scientific induction, belongs to a higher level in the hierarchy of intellectual activity in

comparison with the reproductive work of accountants.[8]
Hence, it is natural that inductive Turing machines can do much more than Turing machines and this feature of inductive Turing machines is mathematically proved.[9] In particular, inductive Turing machines can solve problems that cannot be solved by any Turing machine.
As a supportive evidence (not a proof) for the above statement, it is possible to take what Kurt Gödel wrote in a note entitled "A philosophical error in Turing's work":

> *"Turing gives an argument which is supposed to show that mental procedures cannot go beyond mechanical procedures. However, this argument is inconclusive. What Turing disregards completely is the fact that mind, in its use, is not static, but constantly developing, i.e., we understand abstract terms more and more precisely as we go on using them... though at each stage the number and precision of the abstract terms at our disposal may be finite, both... may converge toward infinity…"*[10]

Implementing both empirical induction and deduction, inductive Turing machines provide more efficient tools for artificial intelligence (AI) in comparison with Turing machines or other recursive algorithms.[11] This is important because researchers explain that recursive algorithms are not adequate tools for AI.[12] Indeed, inductive Turing machines are much more powerful than Turing machines.[13] While Turing machines generate only two lowest levels of the infinite arithmetical hierarchy used as measurement tool of the power of automata and computing machines, inductive Turing machines of higher orders can generate the whole arithmetical hierarchy.[14]
Invention of inductive Turing machines changed the concept of algorithm essentially extending its scope and power.[15] It was a transformation of the computing paradigm in the sense of Kuhn,[16] symbolizing emergence and proliferation of a new type of algorithms called super-recursive algorithms.
Later, other classes of abstract automata that also perform inductive computations – inductive cellular automata,[17] inductive evolutionary machines[18] and periodic Turing machines[19] – were constructed.
In addition to the individual level of a scientist, induction is prevalent in science as a whole. It is possible to find a methodological analysis of inductive processes in Kuhn's and Prigogine & Stengers' works.[20]
However, inductive processes are not bounded by the domain of science – they exist in many other areas, including art and literature. For instance, writing about discourse, Paul Ricoeur stresses that there is always a surplus of meaning that goes beyond what objective techniques seek to explain.[21] There is a surplus of meaning because we apply objective techniques to things we already understand as having a possible meaning without fully exhausting that meaning. The meaning of acts of discourse is moreover always open to new interpretations, particularly as time passes and the very context in which interpretation takes place changes. Consequently, we once more come to inductive processes.

The same is true for the true creations of art and literature. For instance, Leonardo da Vinci wrote "Art is never finished, only abandoned." In a similar vein, Pablo Picasso asserted:

> *"To finish a work? To finish a picture? What nonsense! To finish it means to be through with it, to kill it, to rid it of its soul, to give it its final blow the coup de grace for the painter as well as for the picture."*

This naturally implies that, at least, for some artists the process of creation follows inductive footsteps.
Besides, comprehension and understanding of artistic creations is a kind of discourse and creations of art and literature, especially profound ones, are always open to new interpretations. Culture is changing. Knowledge of people is changing. It brings necessity in new and new interpretations and commentaries. Consequently, we once more come to inductive processes.
In addition, natural languages and especially languages of art convey more than a single meaning. Thus, text in these languages can always be understood in more than one way.

Hence, texts regularly need to be interpreted and reinterpreted while interpretation is an inductive process by its very nature.

As it was already exhibited, even creation of insightful works in art and literature involves inductive processes. For instance, La Rochefoucauld had difficulties arranging his maxims. He issued no fewer than five editions in his lifetime, all with significant alterations, deletions, and addenda. In the fifth edition of 1678, his publisher apologetically notes, "*As for the order of these reflections, you will easily appreciate that it was difficult to arrange them in any order, because all of them deal with different subjects*."[22]
Even more explicitly this trait was expressed when in his conversations with the Russian Music Professor Aleksandr Borisovich Goldenveizer, Leo Tolstoy characterized the creative work of an artist saying:

> *"I can't understand how anyone can write without rewriting everything over and over again. I scarcely ever re-read my published writings, but if by chance I come across a page, it always strikes me: All this must be rewritten; this is how I should have written it."*

This distinctly demonstrates that for Tolstoy writing was an inductive process.
Thus, we can see that inductive processes pervade in all kinds of creative human activities while mathematically they are modeled by inductive Turing machines.

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