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Vortex knots in physics

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Different physical systems share similar state equations and thus, similar solutions. This is the case for gravity, electricity, magnetism, and approximately for elasticity and hydrodynamics, in all cases because of 3D symmetry rules. Thus, all solutions of these static or dynamic problems share the evidence for 1D vortex like singularities. Large samples of such materials exhibit vortex networks. All these vortex networks are quite sensitive to sample thermal history. The simple case of magnetism enables us to derive a numerical study of this magnetic structure. Simple geometric considerations on independent vortices prove that vortex thickness keeps them independent and forbids any branching. Experimental comparisons with simple physical systems evidence for vortex knots. The next step for larger samples consists of knots of vortex knots and so on in a more and more complex structure.

Basic symmetry rules for 3D problems involve equivalence between all directions and senses. This symmetry also occurs for time sense. So basic Maxwell's equations for electricity and magnetism in vacuum read:[1]

$$\frac{\partial^2 g}{\partial t^2} = \frac{L^2}{T^2}\left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}\right) = c^2\left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}\right) \quad (1)$$

In these famous equations, g is any component of the electric field or of the magnetic field. From (1) a local static singularity, i.e., a Dirac electric or magnetic monopole, induces the classical 3D Coulomb field. Therefore, Coulomb electric field is of the same nature as the Newton gravitational field with the same r^{-2} law. For static magnetic dipoles or electric dipoles, equation (1) generates the dipolar interaction; this complex r^{-3} law depends also upon mutual dipole directions. These simple laws are exactly similar to Newton's law for gravitation, here for dipoles, with of course the same solutions and similar structures.

The basic equations for elasticity[2] and hydrodynamics[3] share the same symmetry rules, but are more complex than equation (1) since more phenomenological constants than mass or charge only are involved in these interactions. Instead of mass, electric charge or magnetic moment, both an elasticity modulus and a torsion modulus occur in the equations of elasticity and fluid dynamics. So, the basic elasticity or hydrodynamic equations are of higher order than Coulomb's and Newton's equations. Anyway, in a first approach, there is a strong similarity between all these problems and their solutions are very similar with evidence for similar long ranged interactions. This feature explains the abundance of vortex of quite different sorts in nature. In magnetism for instance this vortex structure is used for memory systems, while recently, skyrmions,[4] with a rather similar structure but a different origin, i.e., spin orbit coupling, and a smaller size, have found new applications.

These introductory words introduce the goals of this paper. A first goal consists in solving the magnetic structure of a basic magnetic sample where exchange stabilizes magnetism, as it often occurs experimentally. Dipolar interactions of relative intensity *d* as compared to unit exchange are active. Exchange is a local interaction, here just between nearest neighbors, while dipolar interaction extends over the whole sample in such a way that all magnetic moments interact together. The competition between exchange and dipolar interactions, as well as the extended nature of dipolar interaction leads to complexity, i.e., complex arrangements of magnetic dipoles, here called spins. Therefore, the magnetic structure depends on both the size and shape of the sample. The other goals of this paper are the extension of these results to large samples and to similar problems such as that of electricity, elasticity and so on.

The basic structure

An extended interaction like the dipolar one is cumulative. As a conclusion of this competition, the optimal magnetic structure remains sensitive to the sample size *L*, a main parameter of this problem, as well as it remains sensitive to the sample shape. Finally, with such a complex problem, many metastable states occur for a given sample, i.e., many different configurations remain stable at low temperature and share quite similar energy levels. Therefore, the actual magnetic structure of a sample also depends on its history and more especially on its thermal history since heating enables to erase past configurations in an annealing process at a temperature *T* that corresponds to a fraction of the exchange energy.

Such complex problems lead to complex solutions and thus, require the use of numerical methods, here Monte-Carlo simulations[5] and Langevin dynamics simulations,[6] in order to observe the large number of approximate solutions. Langevin dynamics have the advantage of introducing dynamical properties of these complex structures with potential applications. Numerous authors[7] use micromagnetic simulation programs such as Mumax with similar results. Experimentally different techniques reveal these complex 3D structures in nano-magnetic samples. This is the case of Lorentz electron microscopy[8] and recently X-ray tomography.[9]

The occurrence of complex vortex structures and at a larger scale of a network of imbricated vortices comes from basic numerical evidence, in agreement with observation in magnetic nanostructures. A main point to underline is the vortex independence that occurs since thick vortices remain separate as observed numerically.

As noticed before, magnetic structures depend upon sample shape and size. Thus, there is a full “zoology” of such magnetic structures as a function of size and shape. Here in this first general approach, we consider rather small sample sizes. In addition we try to extend our results to large 3D samples. For large samples, dipolar effects are quite strong. So considering smaller ones with higher *d* values enables us to approximate large sample structures. Similarly, a cubic sample with simple cubic crystalline structure sounds to be a good first approach of large 3D samples with this basic shape. In our numerical simulations, the cubic sample size *L* is 64 for a 64*64*64 cube, with a simple cubic structure. A magnetic structure means 262144unit vectors. Such a complex magnetic structure is very rich.

In order to give a realistic view of the magnetic configuration, we show in Figure 1 a basic cut at the level z=9 of the low temperature structure found for d=0.1, where the vertical component of the magnetic vector is reported by colors, while the in-plane components of magnetic vectors are shown as in-plane vectors. Several features clearly appear. There are 4 vortex centers where the magnetic vector

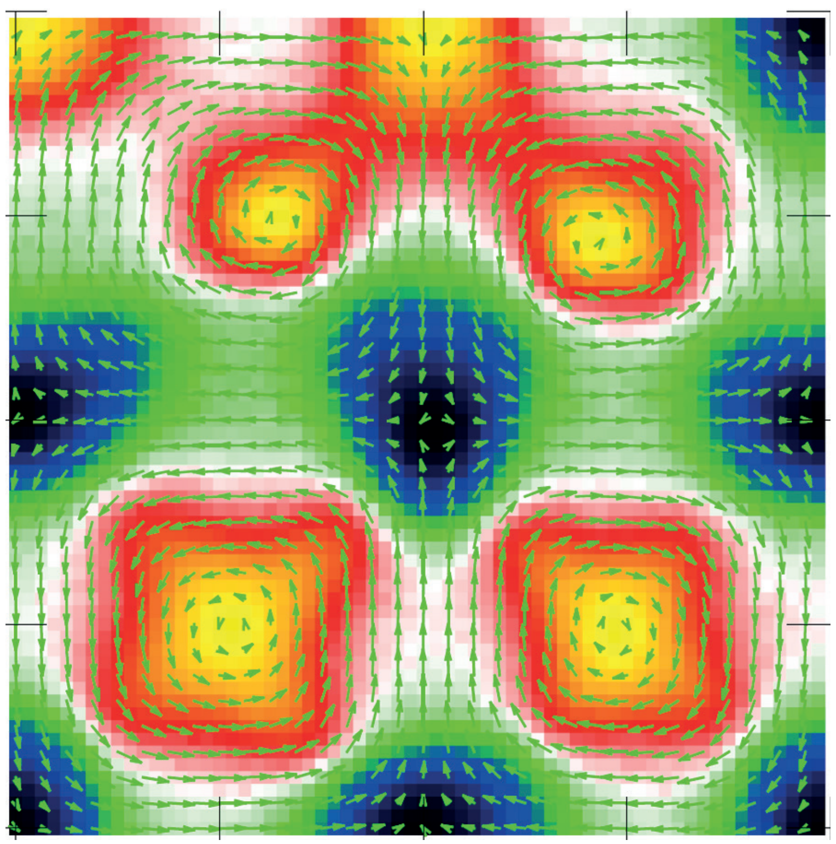

Fig. 1. View of a cut of a metastable magnetic configuration at *z=9, d=0.1 T=0.001*

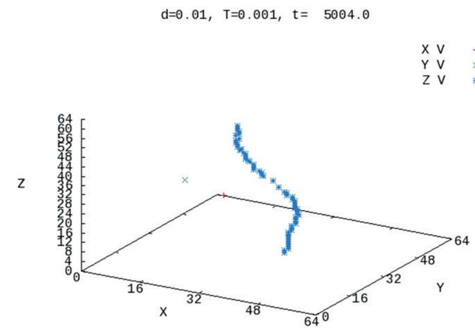

Fig. 2a. d=0.005

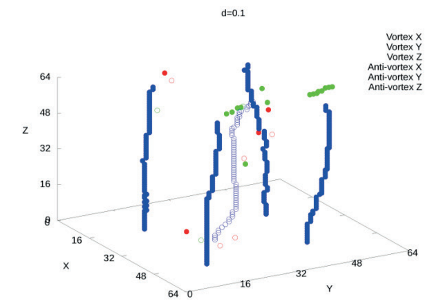

Fig. 2b. d=0.1

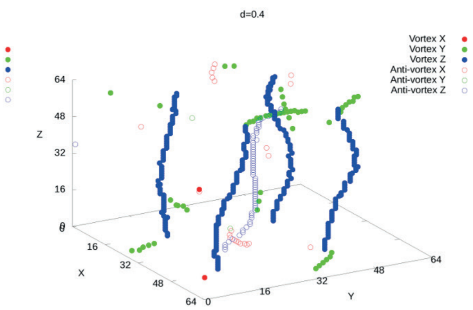

Fig. 2c. d=0.4.

turns clockwise or oppositely. Two close vortices have opposite chirality. Antivortex where magnetic vectors share hyperbolic draws occurs at the center as well as on the side centers. There are also several nearly uniform domains. All vortices and antivortices are rather thick, with the same size as that of regular domains which means the existence of an effective repulsion between such singularities.

So, the next step of representation consists in focusing on singularities only, i.e., on vortices and antivortices neglecting other details. Low temperature results[10] shown in Figure 2 occur for different d values. These Figures with increasing d values correspond to samples with increased sizes and so to samples with an increasing number of vortex lines. During the steps of this expansion, the freedom of vortex lines also increases progressively. Vortex curvature and twist appear quite generally in these starting vortex networks.

More precisely, according to the work reported in reference 10, for *d*=0.005, there is just a single vortex line as shown in Fig. 2a. For *d*=0.01, there is still a single vortex line, which is very mobile, as it must occur close to a phase transition. For *d*=0.02, there are already 4 vortex lines which are not so mobile. For *d*=0.04, 4 vortex lines are rather stiff and for *d*=0.06, they are quite stiff. For *d*=0.1 one antivortex line appears between the 4 vortex lines as seen on Fig. 2b and the global result is more isotropic. For *d*=0.2, the antivortex line is rather stiff. For *d*=0.3, new antivortex lines appear. The further studied steps are *d*=0.4 shown in Fig. 2c and *d*=0.5 where the vortex network is stiffer. More generally, a lot of successive phase transitions provides more and more complex networks and higher and higher symmetry levels. Such complex networks require also higher computational times.

Generalization to complex vortex networks

We use an experimental approach of vortex networks here in order to deal with general optimal arrangements of 1D lines. The characteristics of our magnetic vortices are their effective thickness that forbids them from any branching. The 3 D symmetry of the global arrangement is a requirement. Within the sample volume, several thick curved and twisted lines must appear. In other words, the pseudo vortex lines must be quite longer than the sample size in order to obtain a realistic vortex network.

All these conditions, symmetry, large length in front of the sample size, appear in a washing machine at the end of a full washing process, when washing together several winter shirts, examples of singularity lines. The diameter of the washing drum is about .5 m while the extended length of winter shirts is more than 1.5 m. This ensures the required curvature of vortex lines. The long motion ensures global symmetry. A typical result appears in Figure 3 with the result of more than ten winter shirts. Notice the numerous knots between different

Figure 3. The intermixing of 10 winter shirts at the end of a washing machine process.

shirtsleeves. Such an everyday easy experiment does not work with summer shirts with nearly .5 m length, which are not enough long for inducing strong curvatures.

Quite numerous knots appear between these sleeves as seen on Figure 3. Shirts are highly intricate.

This simple experiment confirms the generality of the recent observations of vortex knots in different fields of physics. For instance, vortex knots appear in the resolution of Navier-Stokes hydrodynamic equations according to a recent paper.[11] Vortex knots also appear in optics[12] and in acoustics[13] according to recent publications.

From this obvious generality of vortex knots, the next research step consists in observing vortex knot networks. Since vortex knots share a central axis, this next step consists of introducing a knot of vortex knots, and so on in a more and more complex structure.

One conclusive remark is devoted to comparisons with experiments using a washing machine. Quite obviously the use of more numerous and longer sleeves will enable us to observe these complex vortex knot networks. Quite similarly, other experiments at a large

scale must provide similar results in other fields

Another conclusion concerns a general structural stiffness effect. The high level of complexity of these vortex networks leads to a strong hardening of these structures. This explains the general experimental requirement of thermal or magnetic bleaching of the samples with complex singularity network. Such a process enables to deal with fresh simple samples without any memory effect. This remark is true in magnetism as well as in elasticity.

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